The laws of logarithms

Introduction

There are a number of rules known as the **laws of logarithms**. These allow expressions involving logarithms to be rewritten in a variety of different ways. The laws apply to logarithms of any base but the same base must be used throughout a calculation.

1. The laws of logarithms

The three main laws are stated here:

First Law
$$\log A + \log B = \log AB$$

This law tells us how to add two logarithms together. Adding $\log A$ and $\log B$ results in the logarithm of the product of A and B, that is $\log AB$.

For example, we can write
$$\log_{10} 5 + \log_{10} 4 = \log_{10}(5 \times 4) = \log_{10} 20$$

The same base, in this case 10, is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

Second Law
$$\log A - \log B = \log \frac{A}{B}$$

So, subtracting $\log B$ from $\log A$ results in $\log \frac{A}{B}$.

For example, we can write
$$\log_e 12 - \log_e 2 = \log_e \frac{12}{2} = \log_e 6$$

The same base, in this case e, is used throughout the calculation. You should verify this by evaluating both sides separately on your calculator.

Third Law
$$\log A^n = n \log A$$

So, for example
$$\log_{10} 5^3 = 3 \log_{10} 5$$

You should verify this by evaluating both sides separately on your calculator.

Two other important results are

$$\log 1 = 0, \qquad \log_m m = 1$$

The logarithm of 1 to any base is always 0, and the logarithm of a number to the same base is always 1. In particular,

$$\log_{10} 10 = 1, \qquad \text{and} \qquad \log_e e = 1$$

Exercises

1. Use the first law to simplify the following.

     a) $\log_{10} 6 + \log_{10} 3$,
     b) $\log x + \log y$,
     c) $\log 4x + \log x$,
     d) $\log a + \log b^2 + \log c^3$.

2. Use the second law to simplify the following.

     a) $\log_{10} 6 - \log_{10} 3$,
     b) $\log x - \log y$,
     c) $\log 4x - \log x$.

3. Use the third law to write each of the following in an alternative form.

     a) $3 \log_{10} 5$,
     b) $2 \log x$,
     c) $\log(4x)^2$,
     d) $5 \ln x^4$,
     e) $\ln 1000$.

4. Simplify $3 \log x - \log x^2$.

Answers

1. a) $\log_{10} 18$,   b) $\log xy$,   c) $\log 4x^2$,   d) $\log ab^2 c^3$.

2. a) $\log_{10} 2$,   b) $\log \frac{x}{y}$,   c) $\log 4$.

3. a) $\log_{10} 5^3$ or $\log_{10} 125$,   b) $\log x^2$,   c) $2 \log(4x)$,   d) $20 \ln x$ or $\ln x^{20}$,
e) $1000 = 10^3$ so $\ln 1000 = 3 \ln 10$.

4. $\log x$.